Pascal's Papers

A puzzle, game and kit about the binomial triangle of numbers discovered and studied in many parts of the world. Called Pascal's triangle in The West, it has other names in other countries. Game cards show something of the history and the other names for this fascinating triangle of numbers. Playing pieces use pathways through the numbers which mathematicians have found to contain beautiful concepts hidden within.

40 min

Game for 2-3 players.
Puzzle requires ablity to add two digit numbers

Table of Contents:

Published by Brush with Science

www.brushwithscience.com

This amazing triangle

was discovered and explored in different countries at different times. Much of The West calls it Pascal's Triangle because of an important paper he published which tied previous discoveries together and advanced beyond what earlier mathematicians had done, including the establishment of a new branch of mathematics in The West called "probability theory". However, it is still called by other names in other countries, who take pride in the discoveries found based on the triangle there.

Age

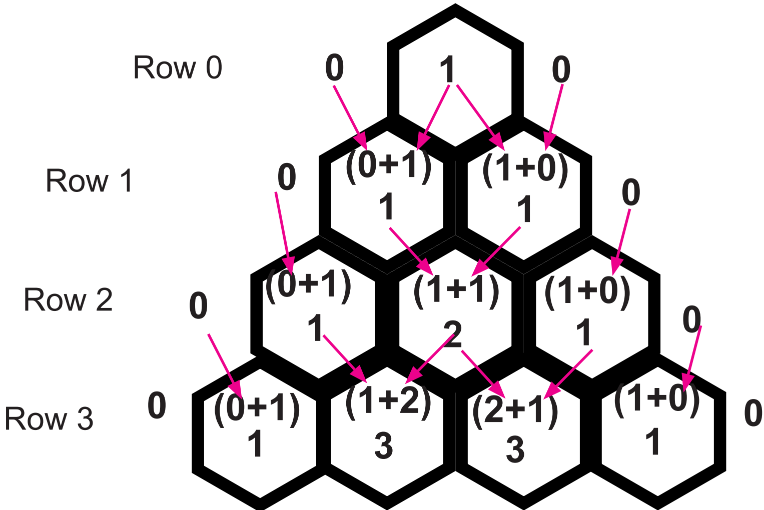
While many concepts that can be explored in Pascal's Triangle are advanced, the games are designed as an introduction. Players who can add multiple digit numbers, or have access to a calculator, can put the triangle together and play the games.

Components

- Wooden puzzle for constructing Pascal's Triangle.
- 27 wooden game pieces.
- A wooden spinner.
- Two 12 sided dice and one six sided die.
- 19 wooden historical cards.

Creating the Triangle

Before playing any games, you should construct the triangle puzzle using the numbered hexagons, without looking at the solution on the box cover. First construct



the frame, made of pieces that are not hexagons, which lock together. You will fill this frame with the numbered hexagons as described next.

Use the side of the hexagons which match the color of the frame. Starting with a **1** at the top (apex) of the triangle, each number will be added to the number on the hexagon next to it to determine what numbers will be on the row below. The **1** at the top is next to nothing, or zero, so there are two hexagons with **1** below that (Row 1). In fact, the sides on the left and right are all **1*s***. These two **1*s*** are added together to get **2** on Row 2. Going left to right, **zero** and **1** are added to give another **1** at the edge on the row below. **1** and **2** are added to get **3**; **2** and **1** are added to get another **3**; and the last **1** is again added to **zero** to get another **1** on the right edge. This continues as shown above.

Once constructed as a puzzle, the triangle of numbers can be used to create visual patterns of numbers, solve mathematical equations, deduce the powers of 11, calculate the Fibonacci Series, and so much more!

The puzzle stops at the 13th row, but in theory, it could go on forever. Once you have completed the triangle, you are ready to look for patterns and try different experiments flipping some of the tiles over. Try flipping over all the hexagonal tiles that are multiples of three. Is there a pattern? There are a lot of books[1] and websites on the subject of the binomial triangle. The purpose of this kit and game is not to tell you everything, but to give you fun tools and games to explore the binomial triangle. Use books or the internet to find the many discoveries mathematicians have made. Maybe you will find one of your own! Play “Pattern Hangman” on page 15.

All the multiples of seven have been flipped over to the light side, to reveal a triangle within the triangle!

---

1 *Pascal’s Triangle: A Study in Combinations*, by Jason VanBilliard, 2014. ISBN-13: 978-1499730616
Pascal’s Triangle 2nd Edition, by Thomas M. Green and Charles L. Hamberg. CreateSpace, Charleston, SC, 2013; ISBN-13: 978-1479289844
Pascal’s Arithmetical Triangle, by A.W.F. Edwards, The John Hopkins University Press 2002. ISBN 0-8018-6946-3

The Game Pascal's Papers

If you have the Cigar Box version, you will need to know all the moves presented in the game Pascal's Papers, but you will use wooden cards instead of a spinner to play your game, called Binomial Checkers. See the next section for the game related to the Cigar Box version.

Historical Background

All of Pascal's original mathematical papers were thought to have been lost. We know about his mathematical contributions because of the writings of others who borrowed them, read and commented on them, published works and letters. Fairly recently, some of his notes were found on the backs of some papers he wrote about philosophical and religions subjects, called the Pensées manuscript[2]. Some of the drawings inspired the design of Pascal's papers in this game.

Game Story

A famous science museum in France has decided to have an exhibit on the binomial triangle, to show the rich history and why it has the name Pascal's Triangle in France, but other names in other countries. The curator and their team are collecting artifacts from around the world for the exhibit. The Bibliothèque Nationale de France has discovered some new mathematical papers by Pascal that they want in the show. Tragically, there is a private collector who is trying to steal the newly discovered papers of Pascal before the show opens.

---

2 *An unknown mathematical manuscript by Blaise Pascal*, by Dominique Descotes Historia, Mathematica Volume 37, Issue 3, August 2010, Pages 503–534 Contexts, emergence and issues of Cartesian geometry: In honour of Henk Bos's 70th birthday.

Object of Pascal's Papers game

Either the thief or the museum curator and their team will win. The thief wins if he/she can steal all six of Pascal's Papers. The curator and team wins if they can find all the artifacts for the show before the thief steals Pascal's papers, or catch the thief red handed!

Setting up for Pascal's Papers

Inside the wooden box, there is one compartment full of puzzle pieces, which you will not need for the game. Using the lid of the box, which is the game board, set the game up as shown. All the artifact cards are face

The six icons representing Pascal's papers in the game.

down, showing only a number. A triangle consisting of all the multiples of seven on the board is placed on top of the corresponding numbers, representing the Bibliothèque Nationale de France. The newly discovered Pascal's papers are placed in the center of this triangle, one paper per hexagon. The museum team is placed on the 1s along the left and right edge. Put the thief at the highest 1 at the top. There is also a spinner and a pair of twelve sided dice you will use.

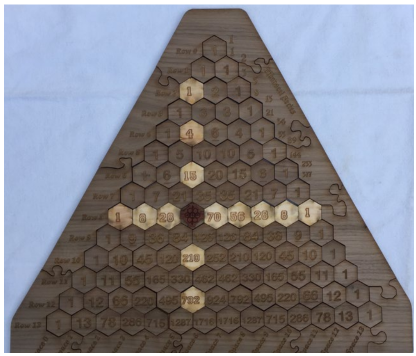
There are 13 white and 13 brown game pieces with symbols on them, which match symbols on the spinner. These are the types of moves you can make, which are inspired by pathways through the triangle mathematicians use.

Moves Explained

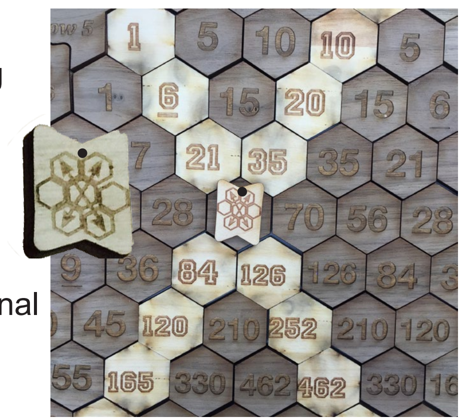
For the first four move types below, you may **not hop over** your own piece or anyone else's, but you may go over the papers and the museum. If you are the thief, you may only land on a team member of the Museum Curator if they are not on their guard post. The thief will then send them to the secret hideout, where they are trapped until the thief is caught or surprised. If you are a part of the Museum Curator team, you may only land on the thief if he/she is carrying a batch of Pascal's Papers.

- **ONE SPACE** allows you to move one space to the tiles closest to you. The light tiles show you where this piece can move:

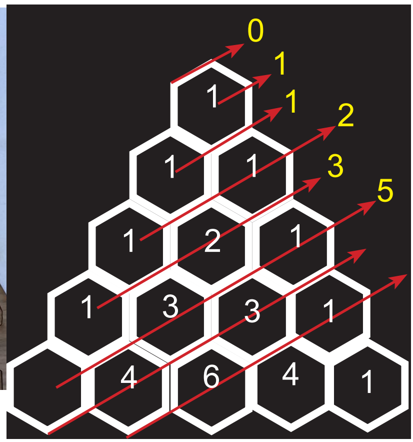
- **ORTHOGONAL** allows you to go side to side or straight up and down as far as you like, or until you run into another piece, including your own. Notice that when going up and down you may move between pieces, because you are not hopping over them.



- **DIAGONAL MOVE** allows you to move along the diagonals as shown as far as you like. A small hole in the top of the piece shows how it goes on the board to help you follow the diagonal lines.

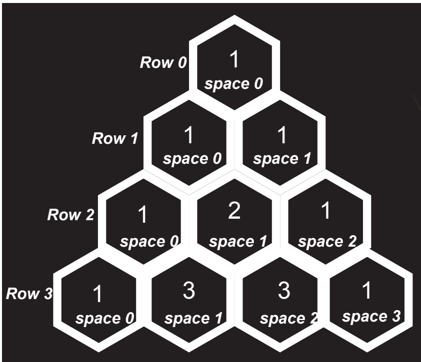


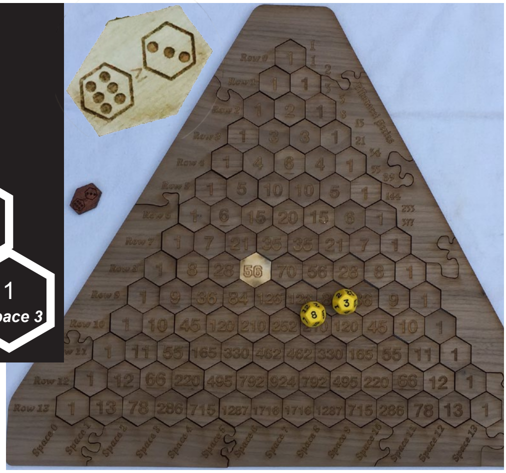
- **FIBONACCI MOVE** is a diagonal that comes out from the four corners of the hexagon on the sides. Sometimes your piece will go between two hexagons and sometimes over a hexagon. It is fine to go between any two pieces, but not to hop over pieces. You could go from lower on the left to upper on the right, as in the picture here, or you could start from the lower right and go up to the left. A small hole at the top of the piece helps orient it on the board, so you can follow the arrows.



The numbers on these diagonals add up to members of the Fibonacci number series. In this series, each number is equal to the sum of the two numbers before it. The first few numbers in the series are 1, 1, 2, 3, 5, 8. 1+1 is 2, 2 + 1 is 3, 3 + 2 is 5, and 5 + 3 is 8. . .

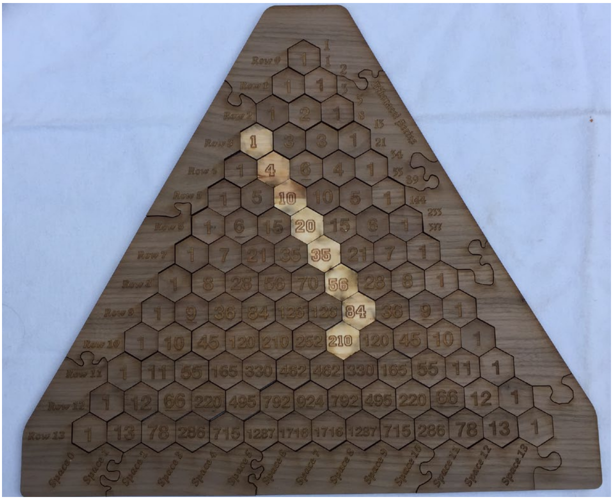
- **DICE CHOOSE** requires you to roll the two twelve sided dice. The larger number is the Row and the smaller number is the Space number in that row you will land on. For example, if you roll an 8 and a 3, you would end up on the tile numbered 56 on the left side of the board. Remember that the first row and the first space are considered to be Row 0 and Space 0. You may go over anything to get to the space indicated by this roll.





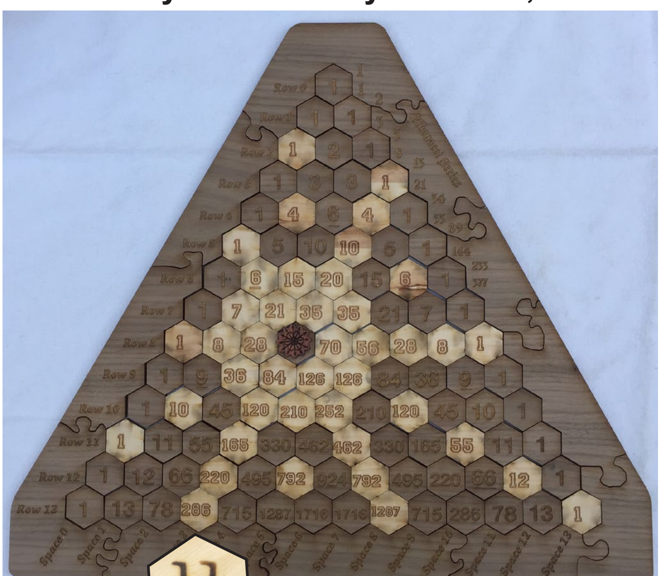
- *This move was inspired by Pascal's study of "combinatorics". It turns out that for any row, say row eight, each space tells you something about how many combinations you can make of eight different things. If you had eight colored stones and placed them in a bag, space 0 tells you how many ways you can pull out zero stones. Space 1 tells you how many ways you can pull out one at a time. There are eight, so if you pull out one at a time, you will do this eight times. Space 2 tells you how many different color combinations you can make by pulling out two stones at a time. This would be 28. Try it*!

- **PASCAL'S HOCKEY STICK** **Use a piece that is currently on a "1"**. Travel in a diagonal line as far as you like and still remain on the board, allowing for at least one row below. Take a little jog in the opposite direction to the hexagonal tile below, which will be your stopping place. This creates a "hockey stick" shape. There is no specific museum team piece for this. You may use any piece that is on a 1 if you spin this move. Also, you may hop over any piece in your way to get to your final space.



Pascal discovered that if you start on a “1”, and move this way, you will land on a number that is the sum of all the numbers in the row you traverse down to get to the final space. Add up all the numbers in the diagonal pictured, and they will equal 210!

- **COMPASS** **As far as you like in any direction,** which includes the Fibonacci move, Diagonal move, and Orthogonal move.



Setting up the pieces

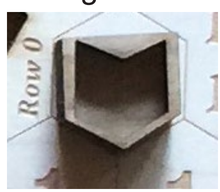
On the left side, the white pieces are arranged as pictured on the left, with two of the “One Space” pieces at the bottom in rows 12 and 13.

On the right side of the board, the brown pieces have two of the “One Space” pieces at the top in rows two and three. This staggers the important moves, much like using the black and white squares when setting up a chess board.

The Dice Choose piece is a very special piece. You can think of it as a Museum Guard. It has special powers when it lands on pieces.

Starting The Game

One person is the Museum Curator and one person is the thief. The thief goes first. If you can not decide who is the thief, roll a 12 sided die. Person with the lowest roll is the thief. If you are the thief, move your piece using the outcome of a spin on the spinner.

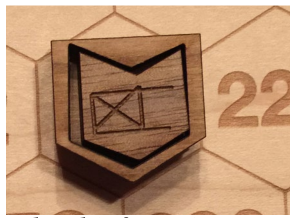


Turn One - the Thief

The thief starts on the number 1 at the top.
Depending on which icon the spinner lands on, the thief

makes a move.

- If the thief lands on the Fibonacci move, there is no where to go when on space 1. Their turn is over.
- If the thief spins Pascal's Hockey Stick, they may travel as far down the row of ones on either side as desired, and jog down to the number that is equal to all of the ones passed by added up. At any other time in the game, if the thief spins a Pascal's Hockey Stick, and the thief is not on a 1 (therefore can not make a move), the thief may spin again.
- The thief must always make some sort of move if possible. This includes the Dice Choose move.
- If the thief lands on a museum team piece, the thief captures the piece and takes it off of the board for the rest of the game
- If the thief lands on a paper with a move other than Dice Choose, thief **gets another turn**. Once the thief is in possession of papers, you can be caught. The goal is to get back to the top space numbered "1" in order to capture this paper. However, it is OK to capture more than one paper before heading back. After each new paper slipped into your pocket, thief gets another turn.



The thief carring one of Pascal's papers.

- If you get Dice Choose, roll the 12 sided dice and move as outlined in the "Moves" section. Here are some possible scenarios for the *Dice Choose* move:

1. If you land on a museum team piece, even if they are in

the Bibliothèque Nationale de France, the raised wooden area in the center, you may take their spot and capture them, taking them off the board.

2. If you land on one of the Pascal's Papers, you get to take this paper straight to your hideout and go back to your number 1 home base at the top. You do not get another turn in this case.

Turn Two - the museum team is next

If three people are playing, the player to the left of the thief will go, in a clockwise manner. One player will control the white team pieces, and the other the brown pieces. If only two people are playing, one is the thief and the other controls both sets of museum team pieces. First, spin the spinner and see which piece you can move.

1. See which move you get.
2. Find a piece with this picture on it. If you spin Pascal's Hockey Stick, you may move any piece you like, as long as it is on a "1" at the start of the turn.
3. You may move as far as you like according to your move with the following rules:

- If you get Dice Choose your Move, you will roll the twelve sided dice to see where you will go. **With this move only**, if you land on the thief, you take their place and the thief must go back to home base, the "1" at the top, if the thief is empty handed. If **this piece** catches the thief while carrying one of Pascal's papers, the thief loses and you win the game!
- With all of the other moves, you may not capture or land on the thief unless they are carrying a set of Pascal's papers. **If you catch the thief with papers, the thief must go to jail for three turns, remaining on the "1" at**

the top of the board.

- You may not land on your own piece or one of Pascal's Papers. If you land on your own piece with the Dice Choose move, put the Dice Choose piece on this space and move the piece currently there to any spot you wish. If you would land on one of Pascal's Papers, roll again.
- You must make some sort of a move if you can.
- You may not go through your own piece, unless you are moving a piece with the Pascal's Hockey Stick or Dice Choose moves. In this case, you may go over anything until you land on your desired/rolled spot.
- You may hop over Pascal's papers with any move.
- If you land on a numbered spot which appears on the back of one of the wooden artifacts, flip this artifact over to reveal the historical document on the other side. One artifact may be revealed per turn. Your goal is to flip all of these over before the thief captures all of Pascal's Papers.

The next and subsequent turns

If there are three players, play continues clockwise. Otherwise, it is the thief's turn again. For the rest of the game, additional rules are:

- Thief can not land on or take a museum team piece that is in the Bibliothèque Nationale de France, the raised piece in the center with all the multiples of seven.
- If the thief gets Pascal's Hockey Stick, and is not currently on a "1", thief spins again.
- Some moves allow you to move between pieces, which is not the same as hopping over them, and is allowed.

Winning And Losing The Game

If the thief steals all the papers before the museum team finds and reveals (flips over) all the wooden artifact cards, the thief is the winner. If the museum team flips over all the artifact cards before the thief can steal all six papers, or if they catch the thief carrying a paper with Dice Choose your Move piece (the Museum Guard), then the Museum Curator team wins. We all win if we get to see a great show on the binomial triangle that includes at least some of Pascal's original papers!

Cigar Box Puzzle Binomial Checkers

If you have the Cigar Box version of the Binomial Triangle Puzzle and Game, you have the puzzle, some move cards, some playing pieces and dice - no spinner or board.

Instead of playing the game above, Pascal's Papers, you have the pieces to play Binomial Checkers. This is a game for two. The goal of the game is to get all of your pieces onto the spaces marked with a "1" on the other side of the board before your opponent.

Setup: The white pieces go on one side of the board, while the brown pieces go on the other side, on the hexagons marked with a "1". A player takes one of each piece and puts them behind their back. This player brings their hands in front with a piece in each hand. The other player chooses a hand. Whichever piece is in the hand chosen is the color that goes first.

Place the wooden move cards face down. First player draws a wooden card from the stack. They may move any or their pieces using this move. The moves are explained above, in the Pascal's Papers game. If there is a piece blocking you in, only the Dice Choose and the Pascal's Hockey Stick moves can get you out. This continues until one player has gotten all of their pieces to the other side. The one to get all their pieces to the other side first is the winner!

Game Two: Pattern Hangman

This game is amazingly fun, yet very simple. Put together the binomial triangle puzzle. Turn all the hexagonal tiles up so that the same color is showing. Think of a sequence of numbers and write it down on a piece of paper. This could be multiples of five, of two, of seven, odd numbers, all the triangular numbers[3], the Fibonacci series, etc. The game is easier if you agree ahead of time on all the patterns you can choose from. However, it can be fun to make up your own and see if they can guess!

The other player gets to ask questions about the pattern. They might say, "Is there a seven?" If there is, the other player turns over all of the sevens on the board, so the other font and color is showing. If there are no sevens, place game pieces from the Pascal's Triangle game on all of the sevens, so you know they have been guessed already. There are 26 game pieces, so they will get 26 guesses or less to try and guess your pattern.

At some point, they can make a guess as to what the pattern is. If they are incorrect, they must give you one of the game pieces. If they can guess what your pattern is before they run out of game pieces, they win! Otherwise, you win if they

---

3 Triangular numbers are numbers that can form a triangular shape. These run in a diagonal column. Find the column with 1, 3, 6, 10, 15, 21, 28, etc. and make triangular shapes out of this many dots for each number to check.

can not guess your pattern by then.

The Historical Artifact Cards

Many cultures lay claim to the binomial triangle, calling it different names. People tend to like to find out who did something first, and give them and their culture credit for being the discoverer or the inventor. In mathematics, the story is usually something built up over time, where the current state of things depends on discoveries before it.

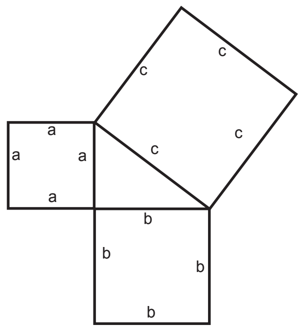
The West gives the binomial triangle the name Pascal's Triangle not because he came up with this particular arrangement of numbers. In his famous paper *Traité du triangle arithmétique*, Pascal starts a new branch of mathematics, the study of probability. He writes about combinatorics and other things, some of which were studied long ago in other cultures, and some that were not. He ties all this together in one powerful paper in a way not done previously, according to the author of ***Pascal's Arithmetical Triangle*** by A.W.F. Edwards.[4] The descriptions of the cards here are no substitute for this fabulous book. They are meant to be a summary and inspire you to look deeper.

The most ancient building blocks of the binomial triangle, according to Pascal in his famous paper, are the figurate numbers. Figurate numbers can be represented by a sequence of

4 Pascal's Arithmetical Triangle, by A.W.F. Edwards, The John Hopkins University Press 2002. ISBN 0-8018-6946-3

evenly spaced points, or a regular geometric arrangement, such as a triangle. Pascal referred back to the work of the Pythagoreans, but you could look back much further. Amir D. Aczel, in his book ***Fermat's Last Theorem: Unlocking the Secret of an Ancient Mathematical Problem***[5], talks about how the ancient Greeks gave credit to the ancient Egyptians as the first to use geometry. Surveyors of land were called "rope stretchers".

The card with the oldest date in this game symbolizes a 3-4-5 triangle made of ropes with knots. Egyptians are said to have used 3-4-5 right triangles when surveying land. In the figure on the right, a = 3, b = 4 and c = 5. A triangle with sides of 3,4, and 5 units follows the formula $a^2 + b^2 = c^2$, or $3^2 + 4^2 = 5^2$ (9 + 16 = 25).



The next wooden card, in decreasing number of years ago, represents the Babylonian "Plimpton" Tablet 322. Neugebauer and Sachs of the American Oriental Society analyzed Plimpton Tablet 322, an ancient Babylonian clay tablet from around 1800 BCE, and discovered that it was a table of Pythagorean triples.

In each row of the tablet, going from right to left, the last column is the row number in the table: 1, 2, 3, 4, etc., written in base 60. It has been translated into our own base 10 system on the card. It has the heading "name" when

5 Published by Four Walls Eight Windows, 1996, ISBN 1568580770, 9781568580777

translated. The third column, when converted from hexadecimal, or base 60, to base 10 notation, would be “z” in the 3-4-5 triangle on the Pythagorean Theorem card below, and has the name “diagonal” when translated. The second column would be “y” and has the name “width”. Since

$$x^2 + y^2 = z^2,$$

$$\text{then } x^2 = z^2 - y^2.$$

In the first row, z = 169 and y = 119. $x^2 = 28561 - 14161$, or $x^2 = 14400$. The square root of 14400 = 120. The first row is not “x”, then, because that would be 120. The first row is z^2 / x^2, or

$$28561 / 14400 = 1.983402.$$

It seems amazing that there is an indication of a “sexagesimal point” (the equivalent of our decimal point for hexadecimal numbers), also called a “radix”, for a mathematical system that is around 3,800 years old![6]

Next in decreasing chronological order is the Pythagorean Theorem wood card. The work of the Pythagoreans was in about 569 BCE. They studied triangular numbers and square numbers,

6 Some of the tablet is missing, but row 14 is complete, and so the pattern can be deduced for the other lines. Although a 1 at the beginning of the number could mean some huge number, it makes more sense that it is a literally a 1, because the calculations for all the rows come out nicely to the decimal shown. There are some “mistakes” in the tablet,and the ancient Babylonians, did not have a symbol for the number “0”.

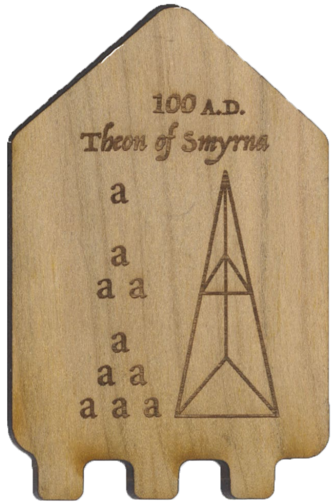
called gnomon, which can be considered different classifications of numbers. This lead to a theory of numbers, according to Edwards.

Going slightly out of order, while discussing Pythagorean triples and the Pythagorean Theorem, the Zhoubi Suanjing (Chou Pei Suan Ching) from 100 BCE and perhaps as much as 1000 years earlier, contains an illustration[7] with a description which is translated “The sum of the squares of lengths of altitude and base is the hypotenuse’s length squared.” This describes what we now call the Pythagorean Theorem, and is more of a proof than the first Ancient Greek version.

While discussing the figurate aspect of the history of the binomial triangle, two more wooden cards from 100 AD should be described. Nicomachas of Gerasa, a Roman province of Syria, and Theon of Smyrna, both described pyramidal or tetrahedral numbers.

7 Source: Chinese Pythagorean theorem, from page 22 of Joseph Needham’s Science and Civilization in China: Volume 3, Mathematics and the Sciences of the Heavens and the Earth, published in 1986 by Cave Books Ltd., based in Taipei (with permission from Cambridge University Press). ISBN:0521058015, from commons.wikimedia.org.

The topic of the next leg of the historical tripod which supports Pascal's Triangle concerns combinations of things, or the combinatorial numbers. How many color

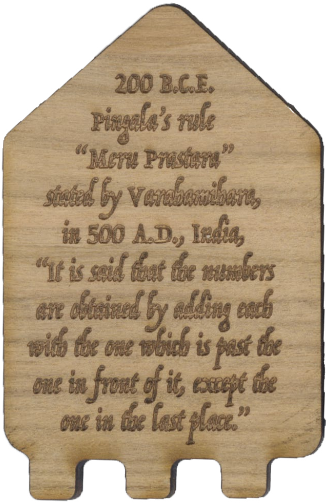


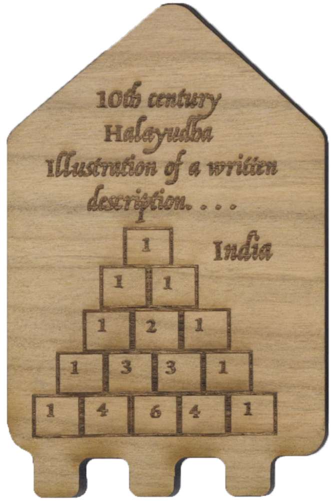
combinations can you make pulling two marbles from a bag containing eight marbles of different colors? How many two letter combinations can you make from the alphabet?

Around 200 BCE, in India, Pingala's rule has been interpreted to be a written description of looking for combinations of things, namely meters of a certain number of syllables[8]. He was looking for all the possible patterns of *n* syllables for any *n*. He gave rules for generating a table of these patterns. The number of combinations possible for one, two, three and four syllable meters of two possible

---

8 Math for Poets and Drummers, Rachel Wells Hall, Department of Mathematics and Computer Science Saint Joseph's University, 5600 City Avenue, Philadelphia, PA 19131 http://www.sju.edu/~rhall rhall@sju.edu, October 31, 2007





sounds gives a pattern. A binomial triangle[9,10] can be used to find the answer to how many combinations one can make. Pingala's work is considered somewhat mysterious. Varahamihara wrote down the rule more clearly in 500 AD, giving credit to Pingala, but did not provide an illustration[11]. An illustrated version was published by Halayudha in the 10th century, who also gave Pingala (or his followers) credit. This early version of the binomial triangle is called "Meru Prastara" in India.

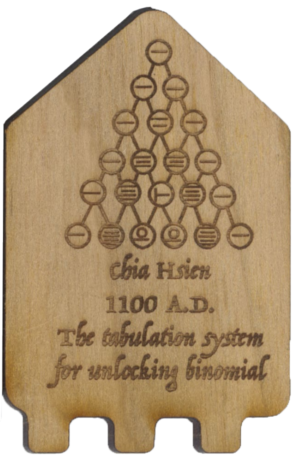
The third leg of the triangle of mathematical subjects invoked by Pascal in his treatise regards binomial coefficients. Over the centuries, some mathematicians figured out that there was a pattern when doing a particular mathematical operation that could be found in the binomial triangle. It has

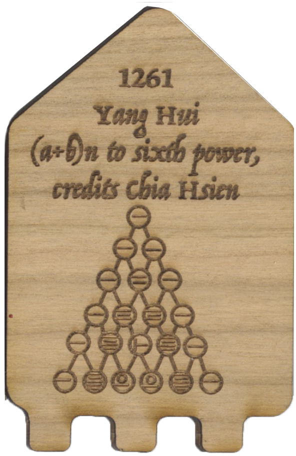
9 Amba Kulkarni. Recursion and combinatorial mathematics in Chandash¯astra, 2007. URL: http://arxiv.org/pdf/math/0703658. Retrieved June 1, 2007.
10 B. Van Nooten. Binary numbers in Indian antiquity. Journal of Indian Philosophy, 21:31–50, 1993.
11 From Edwards, page 30.

to do with two numbers, a and b. If you take a + b, you get 1a + 1 b. The 1 in front of a and b is considered the coefficient, and these are the numbers in row 1 of the triangle, 1 and 1.

To continue this pattern, if you want to multiply a+ b by a + b, or (a+b)(a+b), you get $1a^2 + 2ab + 1b^2$. The coefficient is the number in front of the a's and b's, so in this case, 1, 2, 1. Notice that is the series in the second row of the triangle.

If you continue this way, (a + b)(a + b)(a + b) would be $1a^3 + 3a^2b + 3ab^2 + 1b^3$, or 1,3,3,1 as coefficients; the third row of the triangle.





The next wooden card is from about 1100 AD, by Chia Hsien. Written in Chinese, this triangular arrangement was published again with more rows, to the sixth power by Yang Hui, who credits Chia Hsien, (or Jia Xian). The Chinese call the binomail triangle Chia Hsien's triangle.

Al-Samaw'al, who died in 1180, published a table of binomial coefficients in the work *Al-kitāb al-Bāhir* .[12] This work credits

12 Found in the Istanbul ms. AS2118.

Al-Karaji, for having discovered this in 1007.

Omar Khayyam of Nishapur, Iran, in about 1100, says

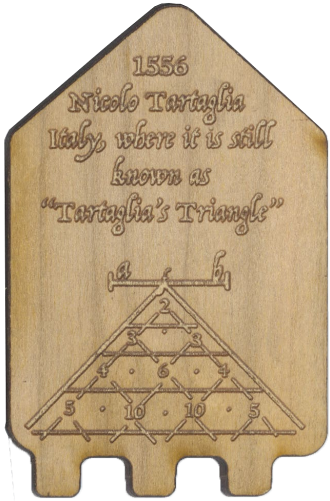
"The Indians have methods of determining the sides of squares and cubes based on some process of induction derived from knowledge of squares of the first nine figures, namely, the squares of one, two, three, etc., and also the product formed by multiplying one number by the next, that is, the product of two and three, etc.

I have composed a book demonstrating the soundness of these methods leading to the discovery of required values and I have added methods for the solution of various other types - I refer to extraction of the sides of the square of the square, the square of the cube, and the cube of the cube, etc. - all of which is new. These proofs are arithmetical, based on the arithmetical part of Euclid's Elements."

In Iran the triangle is called Khayyam's triangle. As the book is missing, the icon has no illustration.[13]

---

13 Daoud S. Kasir, Ph.D. , *The Algebra of Omar Khayyam*, Columbia University Contributions to Education No. 385, Teachers College Series, published by Bureau of Publications, Teachers College, Columbia University, New York City,



In 1225, Jordanus de Nemore[14], who appears to be from Italy based on a translation of his name, published his triangle before the Italian Niccolò Fontana Tartaglia in 1556. Perhaps because Tartaglia published a more extensive study of the triangle, was a well known mathematician in his day, and Jordanus may have remained obscure 300 years after his publication, the binomial triangle is called Tartaglia's triangle in Italy to this day.

Cardano published may books of mathematics, and his own version of the triangle is included on one of the cards. He was a contemporary of Tartaglia, and published one of

---

New York, 1931.

14 Barnabas and Hughs, *The Arithmetical Triangle of Jordanus de Nemore*, Historia Mathematica 16, 213-23, 1989/

Tartaglia's ideas without permission.

German mathematicians Stifel and Scheubelius made their own contributions to the triangle and published versions in around 1500. Cardano's books were sources for some of their conclusions.

Fermat and Pascal wrote letters to each other about how to win games of chance, which are one of the few papers we have giving a glimpse into the thought process Pascal went through when coming up with mathematical concepts.

Probability is an important part of Pascal's treatise, and is embedded in the triangle in interesting ways. The "Dice Choose Your Move" piece in the game was inspired by

Pascal's study of combinatorics.

References / books

Pascal's Triangle: A Study in Combinations, by Jason VanBilliard, 2014. ISBN-13: 978-1499730616

Pascal's Triangle 2nd Edition, by Thomas M. Green and Charles L. Hamberg. CreateSpace, Charleston, SC, 2013; ISBN-13: 978-1479289844

Pascal's Arithmetical Triangle, by A.W.F. Edwards, The John Hopkins University Press 2002. ISBN 0-8018-6946-3

Islam & Mathematics: A Hidden History, a PowerPoint presentation given by Randy K. Schwartz rschwart@schoolcraft.edu as a guest speaker at Macomb Community College (Macomb Co., Mich.) on 2 April 2014, funded by a "Let's Talk About It: Muslim Journeys" grant from the National Endowment for the Humanities and the American Library Association. Found on Academia.edu.

The Algebra of Omar Khayyam, by Daoud S. Kasir, Ph.D., Columbia University Contributions to Education No. 385, Teachers College Series, published by Bureau of Publications, Teachers College, Columbia University, New York City, New York, 1931.

A History of Mathematics, Third Edition, by Uta C. Merzbach and Carl B. Boyer, published by Wiley and sons, Inc., 2011. ISBN 978-0-470-63039-6 (ebk.); ISBN 978-0-470-52548-7 (pbk).

Game Concept and artwork: Julie Newdoll

www.brushwithscience.com www.electronimoes.com


Row 0 1
Row 1 1 1
Row 2 1 2 1
Row 3 1 3 3 1
Row 4 1 4 6 4 1
Row 5 1 5 10 10 5 1
Row 6 1 6 15 20 15 6 1
Row 7 1 7 21 35 35 21 7 1
Row 8 1 8 28 56 70 56 28 8 1
Row 9 1 9 36 84 126 126 84 36 9 1
Row 10 1 10 45 120 210 252 210 120 45 10 1
Row 11 1 11 55 165 330 462 462 330 165 55 11 1
Row 12 1 12 66 220 495 792 924 792 495 220 66 12 1
Row 13 1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
Space 0 Space 1 Space 2 Space 3 Space 4 Space 5 Space 6 Space 7 Space 8 Space 9 Space 10 Space 11 Space 12 Space 13
Fibonacci Series
1 1 2 3 5 8 13 21 34 55 89 144 233 377