Pascal's Papers

Manual

40 min

Game for 2-4 players.
Puzzle requires ablity to
add two digit numbers

Table of Contents:

This game comes in an all wood, handmade version, as well as a printed chip board and paper version. This manual is for the paper version, with a few pictures of the wood version for illustration purposes. On the cover is the wooden game, which was the inspiration and source of graphics for this more affordable paper version.

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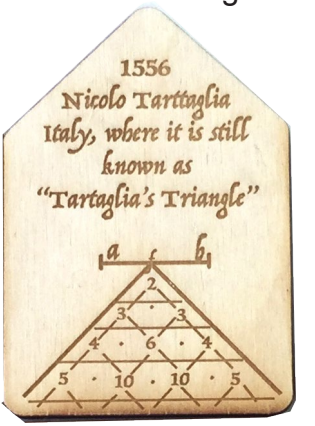
This amazing triangle

was discovered and explored in different countries at different times. Much of The West calls it Pascal's Triangle because of an important paper he published which tied previous discoveries together and advanced beyond what earlier mathematicians had done, including the establishment of a new branch of mathematics in The West called "probability theory". However, it is still called by other names in other countries, who take pride in the discoveries found based on the triangle there.



Age

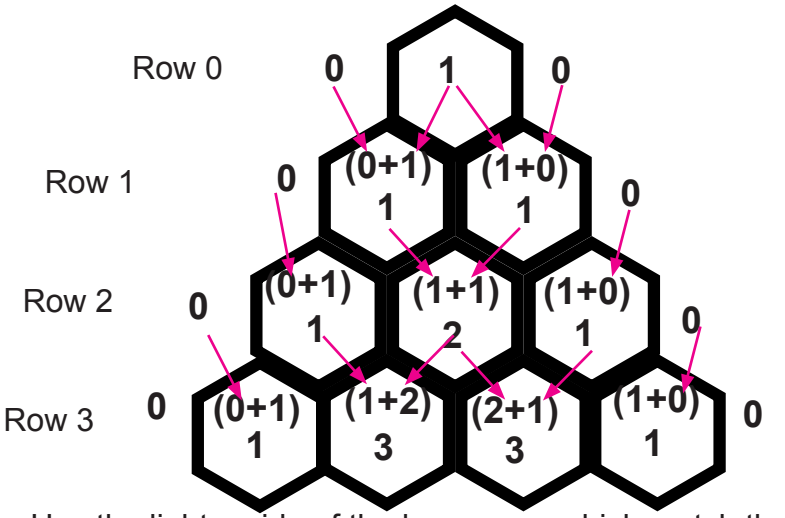
While many concepts that can be explored in Pascal's Triangle are advanced, the games are designed as an introduction. Players who can add multiple digit numbers, or have access to a calculator, can put the triangle together and play the games.

Components

- 102 piece puzzle for constructing the binomial triangle.
- 22 playing pieces.
- A spinner.
- Two 12 sided dice.
- A deck of images of 20 wooden historical cards, plus some instruction cards for the spinner options.
- 29 blank hexagons as replacement pieces and to play pattern games.

Creating the Triangle

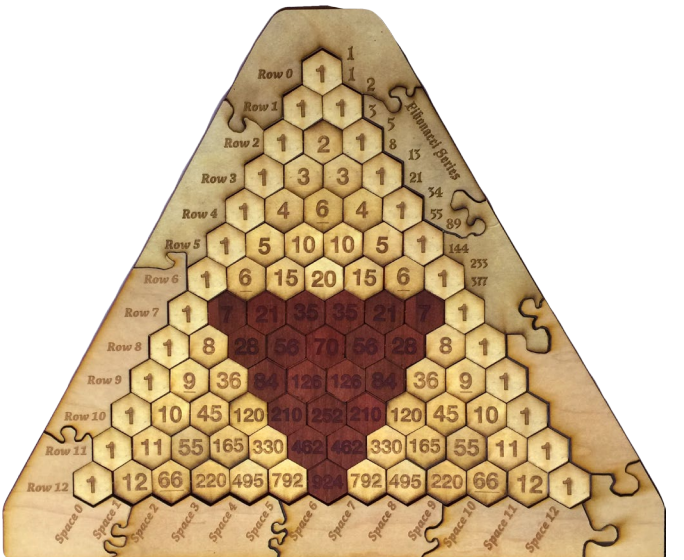
First construct the frame, made of pieces that are not hexagons, which lock together. You will fill this frame with the numbered hexagons.



Use the lighter side of the hexagons, which match the color of the frame. Starting with a **1** at the top (apex) of the triangle, each number will be added to the number on the hexagon next to it to determine what numbers will be on the row below. The **1** at the top is next to nothing, or zero, so there are two hexagons with **1** below that (Row 1). In fact, the sides on the left and right are all **1*s***. These two **1*s*** are added together to get **2** on Row 2. Going left to right, **zero** and **1** are added to give another **1** at the left edge on row three. **1** and **2** are added to get **3**; **2** and **1** are added to get another **3**; and the last **1** is again added to **zero** to get another **1** on the right edge. This continues.

Once constructed as a puzzle, the triangle of numbers can be used to create visual patterns of numbers, solve mathematical equations, deduce the powers of 11, calculate the Fibonacci Series, and so much more! In the wooden version, the puzzle stops at the 13th row, while in the chipboard paper version, it stops at 12. In theory, the puzzle could keep on going forever!

Once completed, try flipping over all the hexagonal tiles that are multiples of three. Is there a pattern? There are a lot of books[1] and websites on the subject of the binomial triangle. The purpose of this kit and game is not to tell you everything, but to give you fun tools and games to explore the binomial triangle. Use books or the internet to find the many discoveries mathematicians have made. Maybe you will find one of your own! Play "Pattern Hangman" on page 15.



All the multiples of seven have been flipped over to the dark side, to reveal a triangle within the triangle!

1 *Pascal's Triangle: A Study in Combinations*, by Jason VanBilliard, 2014. ISBN-13: 978-1499730616
Pascal's Triangle 2nd Edition, by Thomas M. Green and Charles L. Hamberg. CreateSpace, Charleston, SC, 2013; ISBN-13: 978-1479289844
Pascal's Arithmetical Triangle, by A.W.F. Edwards, The John Hopkins University Press 2002. ISBN 0-8018-6946-3

The Game *Pascal's Papers*

Historical Background

All of Pascal's original mathematical papers were thought to have been lost. We know about his mathematical contributions because of the writings of others who borrowed them, read and commented on them; his published works; and a few letters. Fairly recently, some of his notes were found on the backs of some papers he wrote about philosophical and religious subjects, called the Pensées manuscript[2], in the Bibliothèque Nationale de France

Game Story

The Bibliothèque Nationale de France has decided to have an exhibit on the binomial triangle, to show the rich history and why it has the name Pascal's Triangle in France, but other names in other countries. The curator and their team are collecting artifacts from around the world for the exhibit. Tragically, there is a private collector who is trying to steal the newly discovered papers of Pascal before the show opens. The race is on between the thief and the library curator and their staff for Pascal's papers!

Object of Pascal's Papers game

Either the thief (thieves if there are four players) or the library curator and their museum team will win. The thief wins if he/she can steal all six of Pascal's Papers. The curator and team wins if they can find all the artifacts for the show before Pascal's papers are all stolen, or the Museum Guard can catch the thief red handed!

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2 *An unknown mathematical manuscript by Blaise Pascal*, by Dominique Descotes Historia, Mathematica Volume 37, Issue 3, August 2010, Pages 503–534 Contexts, emergence and issues of Cartesian geometry: In honour of Henk Bos's 70th birthday.

Setting up for Pascal's Papers

First you must put together the puzzle as described in the first section. Flip over all the multiples of seven to the dark side, representing the Bibliothèque Nationale de France. The newly discovered Pascal's papers are placed in the center six hexagons of this triangle, one paper per hexagon. Put a thief at the highest 1 at the top. There is also a spinner and a pair of twelve sided dice you will use. All the artifact cards are placed face down, showing only a number, around the outside of the board.

There are 13 white and 13 brown game pieces with symbols on them, which match symbols on the spinner. These are the library museum team, and are placed on the 1s (ones)

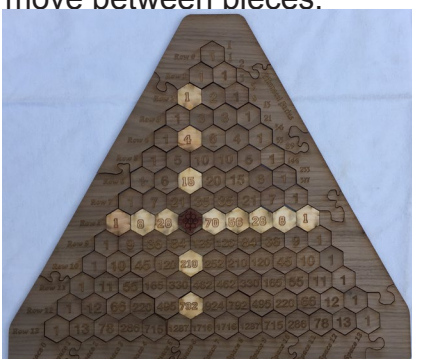
along the left and right edge. These are the types of moves you can make, which are inspired by pathways through the triangle mathematicians use. The remaining cards are instruction cards. They are distributed, one set to the thief (or thieves) and one to the museum team. These are to remind you how to make the moves, explained next.

Moves Explained

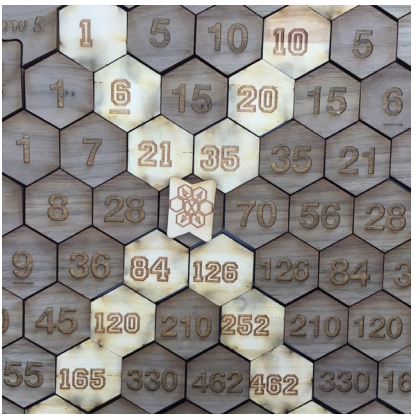
These are summarized on the instruction cards. A more complete description follows.

- ONE SPACE allows you to move one space to the tiles closest to you. The light tiles show you where this piece can move:

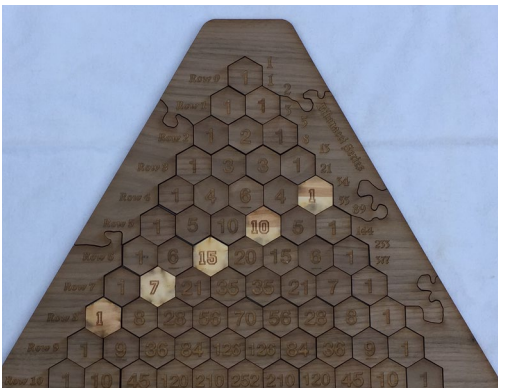
- ORTHOGONAL allows you to go side to side or straight up and down as far as you like, or until you run into another piece, including your own. Notice that when going up and down you may move between pieces, because you are not hopping over them. The black dot is at the top of the piece.



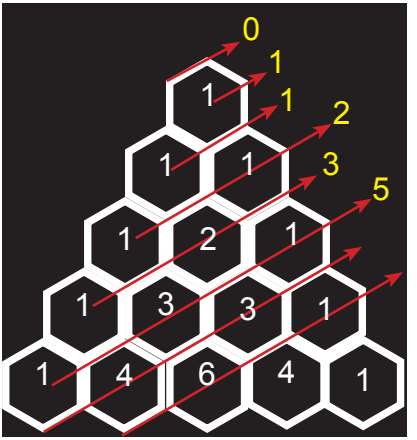
- **DIAGONAL MOVE** allows you to move along the diagonals as shown as far as you like. A small black dot on the top of the piece shows how it goes on the board to help you follow the diagonal lines.



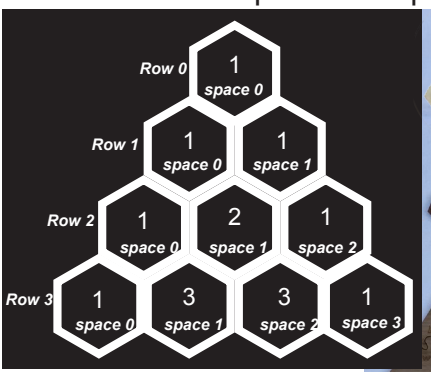
- **FIBONACCI MOVE** is a diagonal that comes out from the four corners of the hexagon on the sides. Sometimes your piece will go between two hexagons and sometimes over a hexagon. It is fine to go between any two pieces, but not to hop over pieces. You could go from lower on the left to upper on the right, as in the picture here, or you could start from the lower right and go up to the left. A small hole at the top of the piece helps orient it on the board, so you can follow the arrows.



The numbers on these diagonals add up to members of the Fibonacci number series. In this series, each number is equal to the sum of the two numbers before it. The first few numbers in the series are 1, 1, 2, 3, 5, 8. 1+1 is 2, 2 + 1 is 3, 3 + 2 is 5, and 5 + 3 is 8. . .



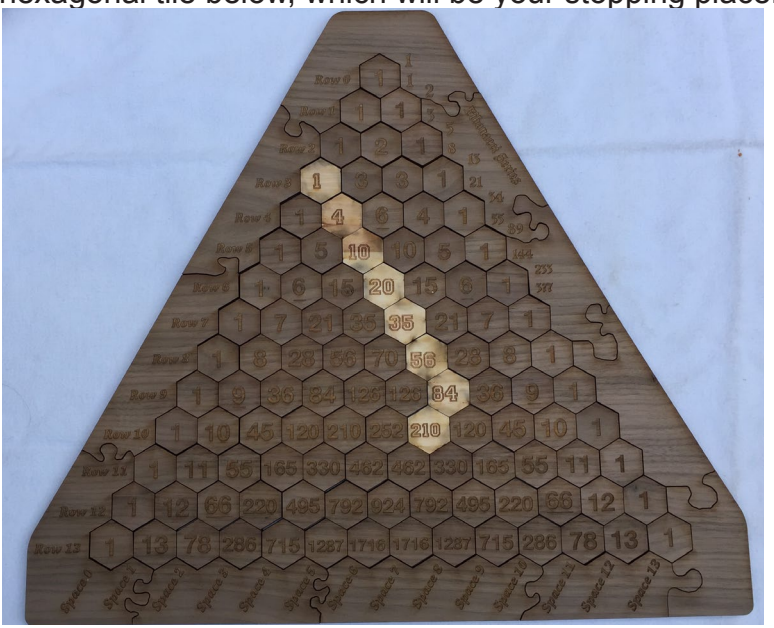
- **DICE CHOOSE** requires you to roll the two twelve sided dice. The larger number is the Row and the smaller number is the Space number in that row you will land on. For example, if you roll an 8 and a 3, you would end up on the tile numbered 56 on the left side of the board. Remember that the first row and the first space are considered to be Row 0 and Space 0. You may go over anything to get to the space indicated by this roll. This move and this piece are special. If the library team lands



on the thief carrying papers using this move, they win! If the thief lands on a library team member, they may capture them even if they are in the museum.

This move was inspired by Pascal's study of "combinatorics". It turns out that for any row, say row eight, each space tells you something about how many combinations you can make of eight different things. If you had eight colored stones and placed them in a bag, space 0 tells you how many ways you can pull out zero stones. Space 1 tells you how many ways you can pull out one at a time. There are eight, so if you pull out one at a time, you will do this eight times. Space 2 tells you how many different color combinations you can make by pulling out two stones at a time. This would be 28. Space 3 tells you how many different color combinations you can make by pulling out three stones at a time, etc. Try it!

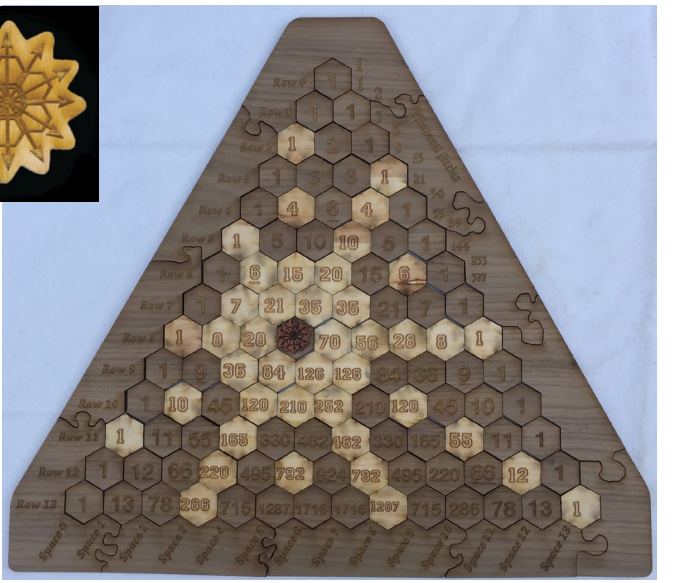
- **PASCAL'S HOCKEY STICK** **Use any piece that is currently on a "1"**. Travel in a diagonal line as far as you like and still remain on the board, allowing for at least one row below. Take a little jog in the opposite direction to the hexagonal tile below, which will be your stopping place.



This creates a “hockey stick” shape (see next page). Also, you may hop over any piece in your way to get to your final space.

Pascal discovered that if you start on a “1”, and move this way, you will land on a number that is the sum of all the numbers in the row you traverse down to get to the final space. Add up all the numbers in the diagonal pictured on the previous page, and they will equal 210!

- COMPASS **As far as you like in any direction,** which includes the Fibonacci move, Diagonal move, and Orthogonal move.



Setting up the pieces

On the right side, the white pieces are arranged starting at row 12, space 11, the “1” on the lowest right corner. Start

with the “Dice Choose” piece and go in this order, bottom to top:

1. Dice Choose
2. One Space
3. Orthogonal
4. One Space
5. Fibonacci
6. One Space
7. Compass
8. One Space
9. Diagonal
10. One Space

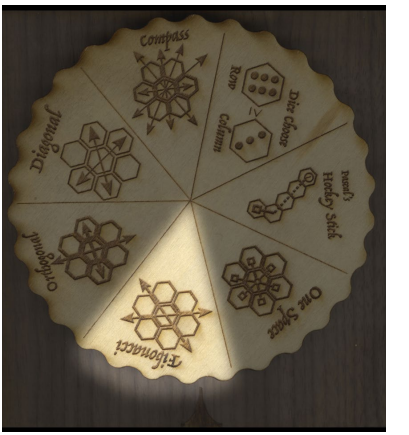
The dark pieces are staggered from the white ones. They start with a One Space piece at the bottom:

1. One Space
2. Dice Choose
3. One Space
4. Orthogonal
5. One Space
6. Fibonacci
7. One Space
8. Compass
9. One Space
10. Diagonal

Starting The Game

One person is the Library Curator and one person is the thief. If there is a third person, they are also a library curator. If there is a fourth, they are the second thief. A thief goes first, and then alternate between thief and museum team. If you can not decide who is the thief, roll a 12 sided die. Person with the lowest roll is the thief. Lowest rolling thief goes first. Spin the spinner to determine your move.

Turn One - the Thief



Thief starts on the number 1 at the top. Depending on which icon the spinner lands on, the thief makes a move.

- If the thief lands on the Fibonacci move, there is no where to go when on space 1 at the top. Their turn is over.
- If the thief spins Pascal's Hockey Stick, since thief begins on a "1", they may travel as far down the row of ones on either side as desired, and jog down to the number that is equal to all of the ones passed by added up, as per the Hockey Stick move described previously. If the thief ever spins a Pascal's Hockey Stick, and the thief is not on a 1 (therefore can not make a move), thief spins again.
- The thief must always make some sort of move if possible. This includes the Dice Choose move.
- If the thief lands on a library team piece, the thief captures the piece and takes it off of the board, UNLESS that piece is in the museum, which means it is on one of the dark multiples of seven.
- If the thief lands on a paper with a move other than Dice

Choose, thief **gets another turn**. Once the thief is in possession of papers, they can be caught. The goal is to get back to the top space numbered "1" in order to capture this paper. However, it is OK to capture more than one paper before heading back. After each new paper is slipped into your pocket, thief gets another turn. Slipping into your pocket simply means stacking the paper on top of your playing piece.

- If you get Dice Choose, roll the 12 sided dice and move as outlined in the "Moves" section. Here are some possible scenarios for the *Dice Choose* move for the thief:

1. If you land on a library team piece, you may take their spot and capture them, taking them off the board, even if they are in the Bibliothèque Nationale de France (one of the dark multiples of seven in the triangular area in the center).
2. If you land on one of the Pascal's Papers, you get to take this paper straight to your hideout and go back to your number 1 home base at the top. You do not get another turn in this case.

Turn Two - the library team is next

If two people are playing, one player is the thief, and the other controls both sets of museum team pieces. If three people are playing, the player to the left of the thief will go, in a clockwise manner, which should be a curator for the museum team. One player will control the white team pieces, and the other the brown pieces. If four are playing, make sure to alternate thieves and museum team turns.

First, spin the spinner and see which piece you move.

1. See which move you get.
2. Find a piece with this picture on it. If you spin Pascal's Hockey Stick, you may move any piece you like, as long as it is on a "1" at the start of the turn.
3. You may move as far as you like according to your move with the following rules:

- If you get **Dice Choose your Move**, you will roll the twelve sided dice to see where you will go. This piece is special. It is considered the Museum Guard.

With this piece only, if you land on an empty handed thief, you take their place and the thief must go back to home base, the "1" at the top, and release back to you the captured piece of your choice.

If you land on the thief carrying a set or more of Pascal's Papers, your Museum Guard has caught the thief red handed! You win the game!!!!!!

If you land on a paper or one of your own pieces, roll again.

- With *all of the other moves*, you may not capture or land on the thief unless they are carrying a set of Pascal's papers. **If you catch the thief with papers, the thief must go to jail for three turns, remaining on the "1" at the top of the board. On their turns, they may try to get out of jail by**

1. **calling "odd" or "even" and rolling the twelve sided die.**
2. **Find the Dice Choose space corresponding to this roll.**

3. **If it is odd, and you called "odds", or if it is even and you called "evens", you get out of jail early and take your turn!**

- You may not land on your own piece or one of Pascal's Papers. If you land on your own or one of Pascal's Papers, roll again.
- You must make some sort of a move if you can.
- You may not go through your own piece, unless you are moving a piece with the Pascal's Hockey Stick or Dice Choose moves. In this case, you may go over anything until you land on your desired/rolled spot.
- You may hop over Pascal's papers with any move.
- If you land on a numbered spot which appears on the back of one of the wooden artifact cards, flip this card over to reveal the historical document on the other side. One artifact may be revealed per turn. Your goal is to flip all of these over before the thief captures all of Pascal's Papers.

The next and subsequent turns

Play continues clockwise. For the rest of the game, additional rules are:

- Thief can not land on or take a library team piece that is in the Bibliothèque Nationale de France, the multiples of seven in the center that have been flipped over to the dark side, unless with the Dice Choose move.
- If the thief gets Pascal's Hockey Stick, and is not currently on a "1", thief spins again.
- If a piece other than the Museum Guard, the Dice Choose piece, lands on the thief with papers in the library, the piece that caught the thief either goes back to the space they came from if it was with the "One Space" piece, or to the "1" space of their choice.

One Space piece

Winning And Losing The Game

If the thief/thieves steals all the papers before the library museum team finds (flips over) all the artifact cards, or is caught red handed by the Museum Guard piece (Dice Chooose), the thief is the winner. Likewise, if the library museum team flips over all the artifact cards before the thief can steal all six papers, or if they catch the thief carrying a paper with Dice Choose your Move piece (the Museum Guard), then the Library Curator team wins.

We all win if we get to see a great show on the binomial triangle that includes at least some of Pascal's original papers! Look at the Artifact Cards supplemental manual for the meanings of the pictures on these cards, and how they are related to the history of the binomial triangle.

Game Two: Pattern Hangman

This game is amazingly fun, yet very simple. Put together the binomial triangle puzzle. Turn all the hexagonal tiles up so that the same color is showing. One person thinks of a sequence of numbers and writes it down on a piece of paper. This could be multiples of five, of two, of seven, odd numbers, all the triangular numbers[3], something to do with the Fibonacci series, etc. The game is easier if you agree ahead of time on all the patterns you can choose from. However, it can be fun to make up your own and see if they can guess!

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3 Triangular numbers are numbers that can form a triangular shape. These run in a diagonal column. Find the column with 1, 3, 6, 10, 15, 21, 28, etc. and make triangular shapes out of this many dots for each number to check.

The other player gets to ask questions about the pattern. They might say, “Is there a seven?” If there is, the other player turns over all of the sevens on the board, so the other font and color is showing. If there are no sevens, place blank hexagons over, or under if you still want to see it, this number wherever it appears on the board. Raised or covered pieces have already been guessed. There are 29 blank tiles, so they will get 29 guesses or less to try and guess your pattern.

At some point, they can make a guess as to what the pattern is. If they are incorrect, they must give you one of the blank tiles. If they can guess what your pattern is before they run out of blank tiles, they win! Otherwise, you win if they can not guess your pattern by then.

Blank tiles may also be used used to replace missing tiles by writing the missing number on the front and back with a bold, waterproof pen.

The Historical Artifact Cards

See the supplemental manual that came with this game, “The Historical Artifact Cards”, to find out about the cards.

The descriptions of the cards in the additional pamphlet “The Historical Artifact Cards”, are no substitute for the fabulous book, ***Pascal’s Arithmetical Triangle*** by A.W.F. Edwards, the source for much of the pamphlet. They are meant to be

a summary and inspire you to look deeper.

References / books

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Game Concept and artwork: Julie Newdoll

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